
REPORT No. 2.

IN TWO PARTS.

INVESTIGATION OF PITOT TUBES.

BY THE UNITED STATES BUREAU OF STANDARDS.

**Part 1.—THE PITOT TUBE AND OTHER ANEMOMETERS FOR
AEROPLANES.**

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Part 2.—THE THEORY OF THE PITOT AND VENTURI TUBES.

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REPORT NO. 2.

PART 1.

THE PITOT TUBE AND OTHER ANEMOMETERS FOR AEROPLANES.

By W. H. HERSCHEL.

1. INTRODUCTION.

The air pressures on the wings of an aeroplane, and therefore the sustaining power of the wings and the stresses to which the whole structure is subject, depend on the speed of the machine relative to the air through which it is moving. The measurement of this speed—particularly near the lower limit where the sustaining power becomes deficient and there is danger of stalling, or at very high speeds where any movement of the controls may give rise to dangerously large stresses—is evidently a matter of importance, and the use of a reliable anemometer or speedometer is highly desirable. The aim of the following paper is to describe the principles of operation of some of the instruments which have been devised or used for this purpose and to discuss their characteristics, so far as it can be done from a general point of view or on the basis of available information, without undertaking new experimental investigations.

Since the Pitot tube is the instrument which has been most commonly used in the United States and Great Britain as a speedometer for aeroplanes, it will be treated first and somewhat more fully than the others.

2. GENERAL REMARKS ON THE PITOT TUBE.

The speed-measuring device known, after its inventor,¹ as the Pitot tube contains two essential elements. The first is the dynamic opening, or mouth of the impact tube, which points directly against the current of liquid or gas of which the speed is to be measured, and receives the impact of the current. The second is the static opening for obtaining the so-called static pressure of the moving fluid, i. e., the pressure which would be indicated by a pressure gauge moving with the current and not subject to impact. To avoid the influence of impact, the static opening points at right angles to the dynamic opening. If the two openings are connected to the two sides of a differential pressure gauge, the gauge shows a head which depends on

¹ Origin and Theory of the Pitot Tube, H. E. Guy Engineering News, June 5, 1913, p. 1172.

the speed and density of the current in which the tube is placed, and which may be used as a measure of the speed of the fluid past the Pitot tube.

If the fluid is a liquid and the two openings are connected to a U gauge containing the same liquid, the gauge shows a head h and the usual formula for computing the speed S is

$$S = C\sqrt{2gh} \quad (1)$$

in which g is the acceleration of gravity and C is the "coefficient" or "constant" of the given instrument. If the head h is read on a gauge containing a liquid of density d while the density of the fluid (either gas or liquid) in which the Pitot tube is immersed is ρ , equation (1) takes the modified form:

$$S = C\sqrt{2g\frac{d}{\rho}h} \quad (2)$$

According to the elementary theory as usually given, C should be exactly 1, and in practice it is in fact in the neighborhood of unity, when the instrument is properly designed and used with suitable precautions.

As regards design, it may be said that numerous recent investigations have shown that almost any sort of dynamic opening is satisfactory, but that the static opening must be designed with great care in order that the coefficient C may be set equal to unity without involving any sensible error in the result of using equation (2). Rowse,¹ for example, has made an extensive comparison of various forms of Pitot tube, which confirms previous results obtained by White,² Taylor,³ Treat,⁴ and others. With the most satisfactory tube tested, the experimental error in S was found to be not over 0.2 per cent. whether the static pressure was taken from a piezometer ring,⁵ or from the static opening of the tube as supplied by the maker. The standard of comparison was a Thomas electric meter, which was assumed to give correct readings.⁶

It may therefore be concluded that by proper construction the Pitot tube can be made to have a coefficient so near unity that for all ordinary purposes the equation

$$S = \sqrt{2g\frac{d}{\rho}h} \quad (3)$$

may be regarded as sensibly accurate.

3. ERRORS WHICH MAY OCCUR IN THE INTERPRETATION OF PITOT-TUBE READINGS.

The simple theory which leads to equation (3) assumes that the tube is always pointed exactly against the current and that the observed head, h , is due to the instantaneous value of the speed S .

¹ W. C. Rowse, Trans. A. S. M. E., 1913, p. 633.

² W. M. White, Journal Association of Engineering Societies, August, 1901.

³ D. W. Taylor, Society of Naval Architects and Marine Engineers, November, 1905.

⁴ Chas. H. Treat, Trans. A. S. M. E., 1913, p. 1019.

⁵ The piezometer was simply an air-tight annular space about the pipe, connected with the interior of the pipe by six small holes.

⁶ For accuracy of Thomas meter see C. C. Thomas, Journal Franklin Institute, vol. 172, p. 411, and Proceedings Am. Gas Inst., vol. 7, 1912, p. 339. For more recent experimental verifications of equation (2) without use of the Thomas meter, see F. H. Bramwell, Report of British Committee on Aeronautics, 1912-1913, p. 35, and Wm. Cramp, Manchester Memoirs, vol. 58, part 2, sec. 7.

These assumptions are never exactly fulfilled in ordinary practice and accordingly exact results may not be obtained, even when no fault is to be found with the instrument itself.

In the first place, it is impossible to read the gauge instantaneously; furthermore, there is always a time lag between the openings and the gauge. Accordingly, even when the current does not change in direction, if its speed varies rapidly all that can be observed is the mean value of h over a certain time interval, and this value does not correspond to the arithmetical mean value of S over the same interval, even if the interval is long compared with the time lag, as has been shown experimentally by Rateau.¹

Disregarding the time lag, the value of S computed by equation (3) will be the root-mean-square speed, which is always larger than the arithmetical mean speed. Hence if, for example, the Pitot tube is being used to determine the discharge through a steam main feeding a reciprocating engine, the computed discharge will be greater than the true discharge. This error is not likely to be very large. If, for instance, the speed varies sinusoidally with time from 0.5 to 1.5 times its arithmetical mean value, the linear speed computed by equation (3) will be 1.0607 times the arithmetical mean speed which determines the total flow, or a trifle over 6 per cent. too large.

A second cause of error is rapid variability in direction of the current, which makes it impossible to keep the tube pointed correctly even when mounted on a vane. If, as is usually the case, it is desired to measure merely the component velocity in a fixed direction, the eddies which almost always exist may introduce a considerable error when this component velocity is computed by equation (3). If the variations of direction are small, the error is due almost entirely to the effect on the static opening and not to change of the direction of impact on the dynamic opening.²

This source of error is much reduced in the Dines tube, a form of Pitot tube in which the static opening consists of a number of round holes or longitudinal slits in a hollow cylinder placed with its axis perpendicular to the direction of the impact tube and to the plane in which the variations of direction are expected to occur. When this instrument is employed as an anemometer, its principal use, the cylinder is of course vertical.

The heads given by the Dines tube are sensibly independent of errors in direction up to about 20° on each side of the mean. To offset this advantage, the instrument is somewhat less sensitive than the ordinary Pitot tube, the coefficient C being greater than 1. Furthermore, each tube must be calibrated separately, and it is not even certain that the coefficient is strictly constant for each tube. Data by Dines³ show a constant coefficient $C=1.53$. Jones and Booth⁴ find values from 1.20 to 1.70 for different tubes. Zahn⁵ finds values from 1.42 to 1.50, depending on the speed.

It has sometimes been doubted whether the coefficient C of a given Pitot tube was dependent solely on the relative speed of the fluid and the tube, the suggestion being that a tube standardized by mov-

¹ Annales des Mines, 1898, p. 341.

² L. F. Moody, Proceedings Engineers' Society of Western Pennsylvania, May, 1914.

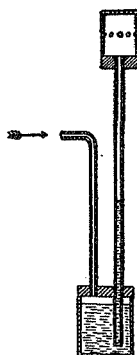
³ Quarterly Journal, Royal Meteorological Society, vol. 18, 1892.

⁴ Aeronautical Journal, July, 1913, p. 195.

⁵ Physical Review, 1903, p. 410.

ing through a quiescent medium, as with a whirling arm in air, may not give correct results when used to determine the velocity of a fluid past a fixed point. It is difficult to see how the Pitot tube can respond to anything but velocity relative to itself. At all events, experiments by Fry and Tyndall¹ have shown that while there was some apparent disagreement at speeds below 11 miles per hour (17.7 kilometers) where the experimental errors were large, for higher speeds, up to 36 miles per hour (58 kilometers) both methods of standardization gave the same result.

Which method of standardization should be adopted—motion of the tube or motion of the fluid—may, nevertheless, depend on the purpose for which the instrument is intended. It is impossible in practice to set up an artificial current of fluid which shall have a high speed and not be turbulent and full of eddies; and the only conditions to which equations (1) and (2) refer are, in strictness, those of steady stream-line flow or steady motion of the tube in a quiescent fluid. If



Dines anemometer.

the tube is to be used in a very turbulent medium, as, for example, in measuring the discharge from a fan, it should be standardized in a stream of fluid in which the turbulence is about the same as it will be under the working conditions. It might very well happen that a given tube when tested on the whirling arm or by moving through still water gave a coefficient $C=1$, while if the tube were tested in a turbulent current some other value of C was obtained. If the tube were to be used to measure the average speed of a similarly turbulent current, this second coefficient should be used and not the value $C=1$.

Apparent errors and inconsistencies in the results obtained by equations (1) and (2) have probably been due in part to disregarding the foregoing obvious considerations.

4. WORKING FORMULAS FOR PERFECT PITOT TUBES.

It will be convenient to collect here, for reference, certain practical working forms of equation (3) for the perfect or ideal Pitot tube, that is, for a tube having the coefficient C equal to unity. If the tube does not satisfy this condition, whether on account of its design or from

¹J. D. Fry and A. M. Tyndall, *Philosophical Magazine* (6), vol. 21, p. 348 1911.

the necessary circumstances of practical use, the value of C must be determined by experiment, and the values of S given by the following equations are then to be multiplied by the observed values of C .

We start by inserting the value $g = 32.17$ ft./sec.² or 9.81 m./sec.² in the general equation (3), viz:

$$S = \sqrt{2gh \frac{d}{\rho}} \quad (3)$$

in which S = the speed of the current,
 h = the head on the differential gauge,
 d = the density of the liquid in the gauge,
 ρ = the density of the current.

From this we obtain special equations for practical use.

(A) *Any two fluids.*— d and ρ may have any values but are to be measured in the same units. The value of S is given by the equation

$$S = X \sqrt{h \frac{d}{\rho}} \quad (4)$$

with the values of X shown in Table 1 for various methods of expressing S and h .

TABLE 1.—Values of X for equation (4).

h measured in—	S measured in—	X .
Inches of liquid of density d	Ft./sec.....	2.316
	Ft./min.....	138.9
	Mile/hour.....	1.579
Mm. of liquid of density d	M./sec.....	.1411
	M./min.....	8.404
	Km./hour.....	.5043

(B) *Any moving fluid, gauge liquid water.*—The value of S is given by the equation

$$S = Y \sqrt{\frac{h}{\rho}} \quad (5)$$

with the values of Y shown in Table 2.

TABLE 2.—Values of Y for equation (5).

h measured in—	ρ measured in—	S measured in—	Y .
Inches of water at 68° F.=20° C	Lbs./ft. ³	Ft./sec.....	18.28
		Ft./min.....	1097
		Mile/hour.....	12.46
Mm. of water at 68° F.=20° C.	Kgm./m. ³	M./sec.....	4.426
		M./min.....	265.5
		Km./hour.....	15.93

When the Pitot tube is to be used in air, the air density ρ for use in equations (4) and (5) may be found as follows:

Let B = the barometric pressure.

Let t = the temperature of the air.

Let P = the pressure of saturated steam at t° , from the steam tables.

Let H = the relative humidity.

Then in English units, if B and P are in inches of mercury and t in degrees F.,

$$\rho = 1.327 \frac{B - 0.376PH}{460 + t} \text{ lbs./ft.}^3 \quad (6)$$

or in metric units, if B and P are in millimeters of mercury and t in degrees C.,

$$\rho = 0.464 \frac{B - 0.376PH}{273 + t} \text{ kgm./m.}^3 \quad (6a)$$

All the numerical data given in this section are accurate enough to permit of computing the speed to within 0.1 per cent. Actual values computed from equation (6) may be found from Table 7, section 13. The calculations required by equation (6) may be avoided by the use of diagrams given by Rowse¹ and Taylor.² Hinz³ gives a diagram showing the gas constant of moist air, which may be used in place of equation (6a).

5. ERRORS OF THE PITOT TUBE AT VERY HIGH SPEEDS.

The theory of the action of the Pitot tube, as given in Part 2 of this paper, shows that the equations given in the preceding sections must be expected to require a correction if the observed pressure difference is enough to compress the fluid sensibly. This will never occur when liquids are in question, though when the instrument is used for measuring the speed of a gas the correction required to allow for compressibility might become sensible at high speeds. But for the highest speeds attained by aeroplanes, say 130 miles per hour, the correction computed from the theory is less than 0.5 per cent., an amount which is altogether negligible in comparison either with the errors of observation or with the uncertainties of the theory itself, which is far from convincingly rigorous.

6. GENERAL REMARKS ON RESISTANCE ANEMOMETERS.

When a fixed obstruction is placed in a current of fluid, it experiences a force in the direction of flow which depends upon and may be used as a measure of the speed of the current. The force depends on the relative motion and is the same, at the same relative speed, when the fluid is at rest and the body moves through it, the force then appearing as a resistance to the motion. It is the resultant of forces exerted on the elements of the surface of the body (*a*) normally by the pressure, which varies from point to point; and (*b*) tangentially

¹ Loc. cit., p. 690.

² Loc. cit., p. 39, and plates 33 and 34.

³ Adolf Hinz, *Thermodynamische Grundlagen der Kolben und Turbokompressoren*, p. 42.

by skin friction of the fluid moving along the surface. Since we are now interested only in devices which may be used as anemometers, we may as well, for the future, say "air" instead of fluid, and "wind" instead of current.

As regards the pressure, there is always, on the windward or upstream side, a region of increased pressure, i. e., of excess above the general static pressure of the air; while on the leeward or downstream side there is a deficiency. In the Pitot tube, the obstruction consists of the impact tube with its open mouth at the upstream end. This receives the excess pressure and transmits it to the gauge. The instrument deals solely with the excess pressure on the upstream side, of an obstruction of particularly simple form, the drag due to skin friction and the suction on the downstream side having no effect on the reading of what we have called a perfect Pitot tube.

The next simplest case is that of a thin flat plate of regular outline set normal to the wind. The skin friction forces balance one another and the whole normal force on the plate is the surface integral of the excess of pressure on the front, over that on the back. If the plate is mounted so that the force of the wind on it can be measured, it constitutes a "pressure-plate anemometer."

Various devices which are in practical use may be regarded as intermediate between the Pitot tube and the pressure plate anemometer. Among these are the Dines tube (see p. 82), the "Stauscheibe," and the Pneumometer. The Stauscheibe is a metal disk about 1 cm. in diameter with holes in the centers of its two faces from which the pressures are led to the two arms of the U gauge, through the disk and through the support by which the disk is held perpendicular to the current. The Pneumometer differs from the Stauscheibe only in details of construction. For both these instruments the coefficient of equation (1) has the value 0.854, the observed pressure difference being influenced by the suction at the downstream face as well as by the impact pressure on the upstream face.¹

In the case of pressure plate anemometers, it is usually the total force acting on the obstruction in the wind that is measured, rather than a manometric pressure, although Stanton² used a diaphragm and air pressure to transmit the force acting on a plate to a manometer 50 feet away.

If the solid obstruction is anything else than a thin flat plate normal to the wind, skin friction as well as pressure contributes to the resultant force; and if the body is not symmetrical about an axis parallel to the wind, the resultant force will not in general be parallel to the wind, but the body will receive a side thrust in addition to the resistance in the direction of the wind, as, for example, when the wing of an aeroplane has both lift and drift. Any body mounted so that the force on it can be measured, provides a means of measuring the speed of the wind and may be used as an anemometer; but if the body is to be held in a fixed orientation with respect to the wind, it is evidently simplest, mechanically, to avoid side thrust by making the body symmetrical about the wind direction, preferably a figure of revolution about that axis. The resistance offered to the wind by a symmetrical body of given maximum section normal to the wind

¹ Rowse, loc. cit., p. 677 and 684. A. Gramberg, *Technische Messungen*, third edition, 1914, p. 99. Cramp, loc. cit., p. 14.

² T. E. Stanton, *Collected Researches*, National Physical Laboratory, Vol. V, 1909, p. 169.

depends greatly on its shape, being less for a sphere than for a flat plate normal to the wind, and still less for a somewhat elongated spindle-shaped body.

Whatever the shape of the body may be, unless it is a sphere its resistance to a given wind depends on its presentation, and by a suitable choice of shape this variation of the force with the orientation may be made quite large. The operation of the Robinson, or cup anemometer, depends on the fact that the resistance of a hemispherical cup is greatest when the concave side is pointed to windward, so that a wind blowing in the plane of rotation of the cups always produces a torque. In the so-called "bridled" form of this anemometer, the torque is measured statically and the instrument is then merely a rather complicated form of pressure-plate anemometer. In the ordinary form of the instrument, in which the cups are allowed to revolve freely, the speed of the wind is measured indirectly by observing the speed of rotation, the action of the wind on the cups being then still more complicated.

From the fact that the pressure recorded by the Pitot tube is proportional to the square of the speed, it might be surmised that the total force observed with a pressure-plate or other static resistance anemometer would probably also be nearly proportional to the square of the speed; and this is confirmed by experiment. The analogy between these anemometers and the Pitot tube is a very close one, the Pitot tube being in principle only a particularly simple kind of resistance anemometer.

We have next to speak somewhat more in detail of some special types of resistance anemometer.

7. THE WIND RESISTANCE OF FLAT PLATES.

The resistance of a flat plate normal to a wind of velocity S is nearly proportional to S^2 and this relation is sometimes represented by writing

$$P = K S^2 \quad (7)$$

in which P is the force per unit area of the plate. The coefficient K is approximately proportional to the density of the air, but it varies with the size and shape of the plate. The independence of Pitot tube readings of the size and nature of the dynamic opening would lead us to expect that the pressure at the center of the front of the plate would be independent of the size and shape of the plate, and Stanton's¹ experiments confirm this expectation. But the suction on the back depends on size as well as speed, thus accounting for the variability of K and showing that P is only a fictitious pressure with no physical significance.

We shall confine our attention to square and round plates, for which the laws of the distribution of pressure are more simple than for very oblong rectangles.² When giving numerical values in "English units" pressure will be in pounds per square foot and speeds

¹ Loc. cit., p. 192.

² G. Finzi and N. Soldati, *Engineering*, Mar. 31, 1905, p. 397.

in miles per hour, while in "Metric units" pressure will be in kilograms per square meter and speeds in meters per second.

A. *Square plates*.—According to Eiffel¹ the value of the coefficient K of equation (7) in English units varies from 0.00266 for plates 4 inches square to $K=0.00326$ for plates 40 inches square or larger. The temperature and pressure of the air during the tests are not given. The corresponding metric values are 0.065 and 0.08. Bairstow and Booth² after analyzing the available data give the equation

$$F=0.00126 (S l)^2+0.0000007 (S l)^3$$

in which F is the total force in pounds, S is the speed in feet per second, and l is the length of side in feet. The equation refers to air at 760 mm. and 15° C. or 59° F. If S is measured in miles per hour the equation becomes

$$F=0.00271(Sl)^2+0.0000022 (Sl)^3$$

and if put into the form (7), for the sake of comparison with Eiffel's results, it may be written

$$P=0.00271(1+0.0008 Sl)S^2$$

the coefficient K depending on both S and l .

B. *Circular disks*.—For a circular disk 30 centimeters, or 11.8 inches, in diameter, Eiffel gives the value $K=0.00276$ English, or 0.0675 metric. Stanton³ found the values $K=0.0027$ English (0.066 metric) by using a 2-inch disk. On the whole, Eiffel's results seem preferable, because the size of disk used by him is more nearly the desirable size for an anemometer.

As regards the relative importance of the front and back of the plate, it may be noted that in a wind of 10 meters per second or 22.4 miles per hour, Eiffel found that the front of his 12-inch disk accounted for 72 per cent of the whole resistance. Zahm⁴ has pointed out that if a plate be surrounded by a sufficiently broad guard ring there will be no suction on the back, while the pressure on the front will be uniform and the same as indicated by a Pitot tube at the same speed.

Table 3 shows the force on a 12-inch disk for different wind velocities, the total resultant force being calculated from Eiffel's value of $K=0.00276$ English (0.0675 metric), and from Bairstow and Booth's formula for square plates, assuming, as some but not all experimenters have found, that the average pressure would be the same for a circular plate with a diameter equal to l , as for a square of side l .

¹ G. Eiffel, *The Resistance of the Air*, p. 35.

² Report, British Advisory Committee for Aeronautics, 1910-11, p. 21.

³ T. E. Stanton, *Proceedings Inst. C. E.*, Vol. CLVI, 1903-4, part 2, p. 78.

⁴ A. F. Zahm, *Journal Franklin Institute*, vol. 173, January-June, 1912, p. 256.

TABLE 3.—Wind forces in pounds on a 12-inch disk.

Wind speed S miles per hour.	Force in pounds according to Eiffel.	Force in pounds according to Bairstow and Booth.
30	1.94	1.97
40	3.47	3.50
50	5.40	5.53
60	7.80	8.00
70	10.60	11.01
80	13.88	14.48
90	17.55	18.48

TABLE 3A.—Wind forces in kilograms on a 30-centimeter disk.

Wind speed S kilometers per hour.	Force in kilo- grams according to Eiffel.	Force in kilo- grams according to Bairstow and Booth.
48.3	0.86	0.87
64.4	1.53	1.55
80.4	2.38	2.44
96.5	3.44	3.53
112.8	4.68	4.87
128.8	6.13	6.39
145.0	7.75	8.15

8. RESISTANCE OF SPHERES AND HEMISPHERES.

Next to thin plates and hemispherical cups the sphere has been most frequently employed in static resistance anemometers as the obstruction opposed to the wind. In addition to the fact that a sphere is symmetrical about all diameters, so that the indications of a sphere anemometer may be made independent of changes in wind direction, the sphere has the further advantage of simplicity of form so that it may readily be duplicated. A disadvantage of the sphere, as compared with thin plates, is the lower value of the coefficient K of equation (7).

According to W. H. Dines, as quoted by Lanchester,¹ K has a value of 0.00154 English for a sphere 6 inches in diameter, or 0.0378 metric for one 153 millimeters in diameter. Dines's tests were made with a velocity of 21 miles an hour (34 kilometers). Eiffel² gives K as 0.00045 (0.011 metric) and explains the difference between his value and that of 0.00112 (0.0275 metric) found at Göttingen, as follows: K decreases with an increase of velocity until a certain critical velocity is reached, after which K remains nearly constant at 0.00045 for the three spheres experimented upon. This critical velocity was found to be about 27 miles an hour for a 6-inch sphere, 16 miles for a 10-inch sphere, and 9 miles for a 13-inch sphere (12,

¹ F. W. Lanchester, *Aerodynamics*, p. 25.² *La Technique Aeronautique*, 1913, p. 146.

7, and 4 meters per second, respectively, for the 16, 24, and 33 centimeter spheres). The high value of the Göttingen coefficient is, according to Eiffel, due to the fact that velocities of over 23 miles an hour (36 kilometers) can not be obtained at that laboratory. It will be noted that even for a 6-inch sphere the critical velocity is well below the lowest flying speeds used in practice.

Table 4 shows values of K for hemispherical cups, according to Dines.

TABLE 4.—Values of K in equation (7) for hemispherical cups.

	English.	Metric.	English.	Metric.	English.	Metric.
	Diameter of cup.					
	2½ in.	64mm.	5in.	127mm.	9in.	229mm.
Cup facing wind.....	0.00597	0.146	0.00386	0.095	0.00402	0.099
Cup with back to wind.....	.00239	.059	.00168	.041	.00138	.034

Since Dines used only the one speed of 21 miles an hour, there is a doubt whether his values would hold for higher speeds. It appears that with a cup there would be little if any reduction in diameter as compared with a plate giving an equal force, though the cup would have the advantage of greater strength for a given force and weight. The difference in the force acting on the cup in its two positions, which is the driving force of the Robinson anemometer, is clearly indicated by the table.

9. PRACTICAL FORMS OF RESISTANCE ANEMOMETER.

Maxim¹ used a pressure plate anemometer consisting of a disk with a spring resistance. His arrangement had the advantage of fairly uniform graduations of the scale, the spring acting indirectly, with variable leverage on the pressure plate.

In the pressure-plate anemometer of Dines² the variable resistance is furnished by a float partly immersed in water, the pressure on the plate being equal to the weight of a volume of water equal to that of the part of the float raised above the water level.

The 1914 catalogue of Aera, Paris, shows a pressure plate anemometer which is merely a speed indicator. It is supplied with three disks, so that it may be set for any speed between 50 and 75 miles an hour (80 and 120 kilometers). The pointer will then show whether the actual speed is above or below the normal. Aera also make an anemometer using a sphere, in the form of a pendulum. This instrument reads only to 45 miles an hour (72 kilometers) and has graduations coming closer together at higher speeds. It would be very inaccurate without some means for holding it vertical.

The Davis Lyall air speed indicator, made by John Davis & Son, of Derby, England, is a bridled anemometer of the screw type which should be held with its back to the wind, though the manufacturers

¹ H. Maxim, *Natural and Artificial Flight*, p. 70.

² *Quarterly Journal*, Royal Meteorological Society, vol. 18, 1892, p. 167.

do not provide it with an air vane to do this automatically. This defect is remedied in the Aera bridled anemometer. Concerning the Davis Lyall instrument, it is stated:

To avoid undue oscillation of the pointer a damper is provided—either magnetic or air. Such a damper is rendered necessary in measuring velocities in a natural wind which varies within wide limits.

When it is desired to investigate the gusty character of natural winds, the sensitiveness of a bridled anemometer becomes an advantage. Concerning a bridled anemometer consisting of five hemispherical cups attached to a vertical spindle by short arms, Stanton¹ says that this instrument is more sensitive to momentary gusts than any of the other recording instruments in common use.

10. THE ANEMO-TACHOMETER.

When anemometers of the screw type are used for high velocities, there is danger that the vanes will be deformed and the velocity indications become unreliable, and for this reason cup anemometers are more suitable for out-door work. Wilhelm Morell, of Leipzig, has placed on the market an anemo-tachometer illustrated in the *Deutsche Luftfahrer*.² This is a Robinson anemometer with tachometer attached for aeronautical purposes, the tachometer being an instrument, usually actuated by centrifugal force like a steam engine flyball governor, so that velocities may be read at a glance from the position of a pointer. It will be noted that with a tachometer, in contrast to a revolution counter, no measurement of a time interval is required. The anemo-tachometer also has the advantage of all Robinson anemometers that the wind vane may be dispensed with.

According to a communication from Morell, his anemometers are calibrated in a wind tunnel, built in accordance with designs of Prof. Prandtl of the University of Göttingen, in which air currents up to 78 miles per hour (125 kilometers), can be obtained. It is stated that some of these instruments have been in constant use for two years without needing recalibration.

The anemo-tachometer, as well as other anemometers, should be attached to the aeroplane in such a manner that its indications are not influenced by the irregular and indeterminate wash of the machine and propeller. It has been proposed to lengthen the distance between the cups and the casing, so as to bring the cups above the upper supporting plane, while keeping the dial on a level with the pilot's line of vision. The objection to this lengthening is that it might change the friction and hence the indications of the instrument, and necessitate a special calibration.

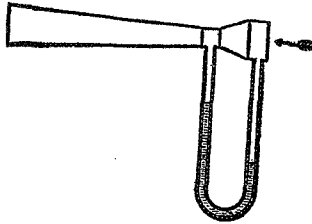
What appears at first sight to be a solution of the difficulty, would be to provide the anemometer axis with a small electric generator, and use the electric voltage, thus generated to indicate speed of rotation by means of a voltmeter. We should anticipate, however, that electric indicating instruments, as at present constructed, would not long retain their accuracy when exposed to the vibrations on an aeroplane.

¹ Collected Researches, National Physical Laboratory, Vol. V, p. 174.

² Apr. 2, 1913, p. 168.

11. THE BOURDON-VENTURI ANEMOMETER.

The Venturi tube consists of a short converging inlet followed by a long diverging cone, the entrance and exit diameters being usually equal so that the tube may be inserted as a section of a pipe line. There is generally a short cylindrical throat. The converging part has somewhat the shape of a vena contracta, but its exact form is of little importance. The exit cone has a total angle of about 5° , this being found to give the minimum frictional loss for a given increase of diameter.



Venturi tube.

When a current of fluid passes through the tube, the pressure in the throat is less than at entrance to the converging inlet, by an amount which depends on the ratio of entrance to throat area, the density of the fluid, and the speed of flow. If the tube is provided with side holes and connections to a differential gauge by which this pressure difference may be observed, it constitutes a Venturi meter. The area ratio is a known constant for a given tube, so that when the density of the fluid is known the observed pressure difference may be used as a measure of the speed of flow. When the pressure difference is expressed as the height of a water column, it is known technically as the "head on Venturi."

Such an instrument may be used as an anemometer by pointing it so that the wind blows directly through it, and the observed head may then serve as a measure of the wind speed. Bourdon¹ employed the Venturi tube for this purpose in 1881, and it has been used recently as an aeroplane anemometer.

At a given speed, the observed head increases with the ratio α of entrance to throat area and the instrument may be made to give a much larger head than a Pitot tube. This is illustrated by the figures given in Table 5 for a tube in which $\alpha = 4$, the throat having half the diameter of the entrance. The data are for air at atmospheric pressure and 70° F. Column (2) gives the head which would be observed with a Pitot tube; column (3) that observed by Bourdon; and column (4) the ratio of (3) to (2).

¹ Annales des Mines, September and October, 1881; Comptes Rendus, 1882, p. 229.

TABLE 5.—Comparison of Pitot and Venturi heads for $\alpha=4$.

(1) Wind speed.		(2) Pitot-tube head.		(3) Head on Venturi according to Bourdon.		(4) Col. 3. Col. 2.	(5) Theoretical head on Venturi.	
<i>Miles hour.</i>	<i>Meters sec.</i>	<i>Ins.</i>	<i>Mm.</i>	<i>Ins.</i>	<i>Mm.</i>		<i>Ins.</i>	<i>Mm.</i>
10	4.47	0.05	1.3	0.17	4	3.4	0.7	18
20	8.94	.19	4.8	.80	20	4.2	2.9	74
30	13.41	.43	10.9	2.30	58	5.3	6.8	173
40	17.88	.77	19.6	4.0*	102*	5.2	12.3	312
50	22.35	1.20	30.5	6.6*	168*	5.5	20.0	508
60	26.82	1.73	43.9	10.0*	254*	5.8	30.0	762
70	31.29	2.35	59.7	15.0*	381*	6.4	45.0	1,143
80	35.76	3.07	78.0	20.0*	508*	6.5	63.0	1,600
90	40.23	3.89	98.8	25.0*	635*	6.4	90.0	2,286

In figure 1 the line HG represents Bourdon's observations and the starred values in column (3) of Table 5 were read from the dotted extension of this curve. While this extrapolation can make no claim to accuracy, it appears from column (4) of Table 5 that a Venturi tube with a 2 to 1 diameter ratio would probably give at least five times as much head as a Pitot tube at ordinary aeroplane speeds.

The curve FE of figure 1 and the numbers in column (5) of Table 5 were found from equation (27) of Part 2, which is known experimentally to agree closely with the facts when the Venturi meter is inserted in a pipe line instead of being used as an anemometer with both ends free. Upon introducing the known values of k and ρ for air at one atmosphere and 70° F., equation (27) reduces to

$$S = 1720 \sqrt{\frac{\frac{10}{r^7} (1 - r^{\frac{2}{7}})}{\alpha^2 - r^{\frac{10}{7}}}} \text{ miles per hour.}$$

If the 1720 is replaced by 769, the result will be in meters per second.

What part of the great discrepancy between columns (3) and (5) of Table 5, or between EF and GH of figure 1, is to be ascribed to friction or other circumstances which make the Venturi tube act differently as an anemometer and as a flow meter, and what part to Bourdon's experimental arrangements and possible errors of observation, can not be decided without further investigation; but in any event, it is obvious that with the Venturi tube a much larger head is available than with a Pitot tube.

Since Bourdon wanted an anemometer for very low speeds, he increased the available head still farther by using two concentric tubes, the exit end of the inner one being at the throat of the outer, so that the suction there increased the speed through the inner tube and the fall of pressure at its throat. The proportions of the tubes which were adopted as giving the best results were as shown in Table 6.

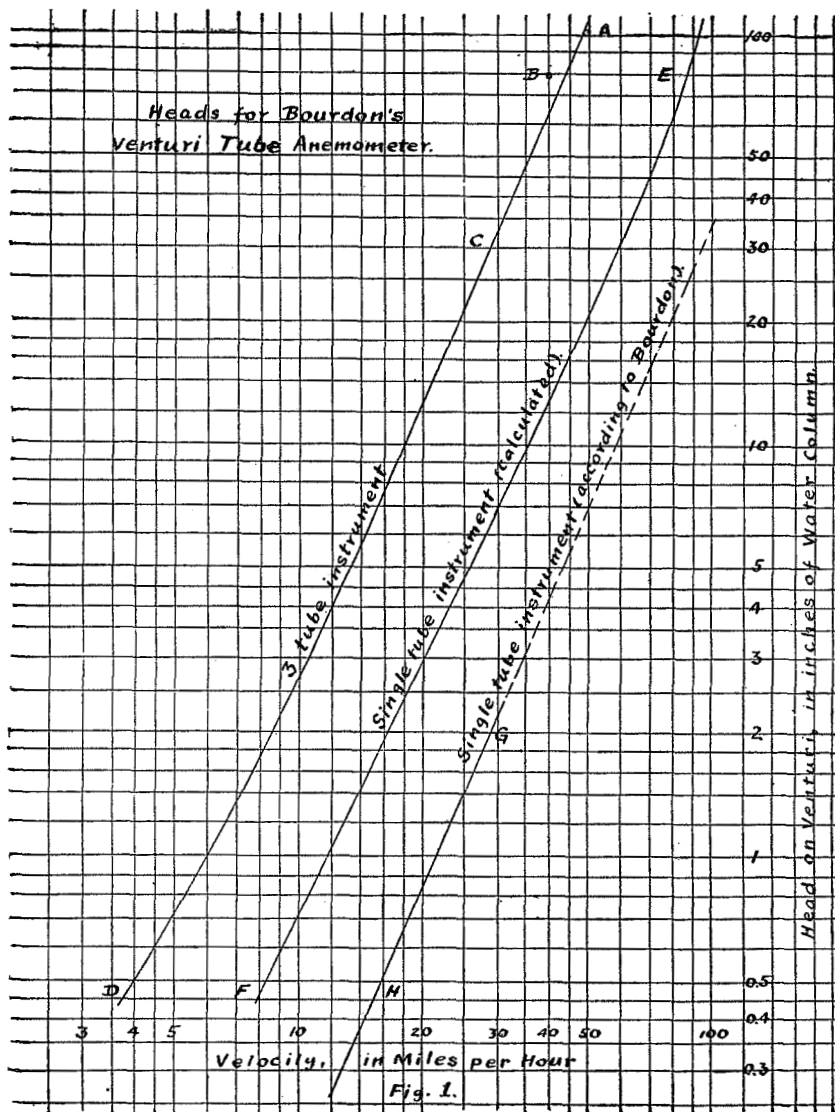


TABLE 6.—Proportions of Bourdon's double Venturi tube anemometer.

	Inner tube.	Outer tube.
Ratio of minimum to maximum diameter:		
(a) Of converging cone.....	0.31	0.56
(b) Of diverging cone.....	0.45	0.60
Double angle:		
(a) Of converging cone.....	34° 15	21° 38
(b) Of diverging cone.....	3° 45	4° 50
Relative throat diameters.....	1.0	6.2

No cylindrical throat piece was used with either tube, the converging and diverging cones being connected directly.

Bourdon also used a similar arrangement of three concentric tubes. The heads obtained with this, at various wind speeds, are shown on figure 1 by the curve *DC* and by the isolated point *A*. The point *B* is from tests of a 3-tube instrument by Brown Boveri & Co.¹

The proportions of single-tube anemometers as used in modern French practice seem to be somewhat like those of Bourdon's inner tube. (See Table 6.) The length of tube in the anemometer made by Aera, of Paris, is 6.3 inches (160 mm.) or nearly the same as the length of the diverging cone of Bourdon's inner tube. Dorand² gives, without dimensions, a section of a Venturi-tube anemometer which indicates a ratio of throat to entrance diameter of about 0.2. The proportions proposed by Toussaint and Lepère³ as a result of recent experiments are very similar to those of Bourdon's outer tube. (See Table 6.)

12. REMARKS ON THE SPECIAL CONDITIONS TO WHICH AEROPLANE ANEMOMETERS ARE SUBJECT.

A. Weight and head resistance.—These must both be small—the smaller the better. Accordingly we need not consider any essentially heavy instruments, such as those which require the use of electric batteries, nor instruments like large pressure plates which offer a head resistance of several pounds.

B. Robustness.—The very severe conditions of vibration preclude the possibility of using instruments which are not mechanically strong or which can not be made so without too great weight. Both the anemometer head proper, and the transmitting and indicating parts must be simple, light, strong, and free from the need of delicate adjustment or frequent testing.

C. Position.—The head must, so far as practicable, be out of reach of irregular currents and eddies and therefore at some distance from the indicator or dial in front of the pilot. The available positions are (*a*) in front of the center of the machine, (*b*) well above the upper planes over the pilot's head, (*c*) near one wing tip. Position (*a*) might be practicable and satisfactory in some cases but there is a possibility, unless the head were very far in front, that the readings might not be the same, at a given speed, during normal flight as when planing with the motor stopped. We have no information on this point. The influence of the body extends some distance ahead, a fact which should not be overlooked.⁴ Position (*b*) would often require the construction of a special support, increasing the weight and head resistance. Position (*c*) seems the natural one to adopt if a transmission of the requisite length can be made satisfactory; but here again it should be noted that the disturbance due to a strut or wing begins some distance ahead of the leading edge.⁵

D. Orientation.—While most anemometers have to be pointed directly into the wind if they are to indicate its resultant velocity,

¹ Zeitschr. d. Ver. Deutscher Ingenieure, 1907, p. 1848.

² E. Dorand, La Technique Aeronautique, Nov. 1, 1911, p. 252.

³ Rep. Brit. Adv. Com. for Aeronautics, 1912-13, p. 396.

⁴ See, for example, the results of experiments on the Marienfelde-Zossen high-speed electric railway, The Electrician, June 17, 1904.

⁵ See E. F. Relf, Rep. Brit. Adv. Com. for Aeronautics, 1912-13, p. 133.

what is needed in aviation is primarily the relative wind speed along a direction fixed with regard to the axis of the machine. The undesirable complication of mounting the anemometer head on a wind vane is therefore unnecessary and the head may be fixed. If information is required about motion perpendicular to this direction, it may be got from a wind vane.

E. *Independence of gravity.*—On account of the very considerable angles of heeling and pitching, it seems useless to consider any instrument which depends for its action on weights or liquid manometers. Any required forces must be applied by springs; or if pressures are to be registered, it must be by spring gauges. Furthermore, all parts of the instrument must be so balanced that the readings are not affected at all by gravity. This remark applies to the transmission and the indicator as well as to the head.

F. *Vertical acceleration and centrifugal force.*—Vertical acceleration acts merely as a change of the intensity of gravity. It will, therefore, have no effect on an instrument which is properly constructed in accordance with E, above.

Centrifugal force must be allowed for in a similar way by careful balancing of all movable parts so that the lateral acceleration of the whole machine during curved flight shall not influence the readings. This balancing in the transmission is equally necessary, whether forces are transmitted by rods or wires or pressures by fluids in tubes.¹

13. DENSITY CORRECTIONS.

Before considering the effects of changes of air density on the indications of particular types of anemometer it will be well to see how great these variations are likely to be under working conditions. For this purpose we consult equation (6) of section 4, viz,

$$\rho = 1.327 \frac{B - 0.376 PH}{460 + t} \quad (6)$$

in which

- ρ = the density of the air in pounds per cubic foot.
- B = the barometric pressure in inches of mercury.
- t = the temperature of the air in degrees Fahrenheit.
- P = the pressure of saturated steam at t° in inches of mercury.
- H = the relative humidity ($H = 1.0$ for saturated air).

The ranges we shall assume are: $B = 30$ to 20 inches, corresponding to a rise from sea level to about 10,000 feet altitude; $t = 0^\circ$ to 90° F.; $H = 0.0$ to 1.0 , i. e., from complete dryness to saturation.

We may first consider the term $0.376 PH$. Taking P from the steam tables we have

at $t = 50^\circ$	70°	90°
$0.376 P = 0.136$	0.278	0.533
$0.376 P \times 0.5 = 0.068$	0.139	0.267

¹ For a discussion of the effect of vertical acceleration and centrifugal force on liquid manometers the reader may be referred to an article by H. Darwin, *Aeronautical Journal*, July, 1913, p. 170.

If we assume a constant relative humidity $H=0.5$, while in fact the humidity varies all the way from 0.0 to 1.0, the maximum error we can make in the value of $0.376 PH$ is $0.376 P \times 0.5$, of which the values at 50° , 70° , and 90° are shown above. To find the percentage error which this assumption can introduce into the computed value of ρ , we must compare these errors with the value of B . The following table shows the maximum per cent. errors in ρ at 50° , 70° , and 90° F. and at 20 and 30 inches pressure which can be caused by assuming $H=0.5$.

	$t=50^\circ$	$t=70^\circ$	$t=90^\circ$
$B=20$ inches	0.34%	0.70%	1.33%
$B=30$ inches	0.23%	0.46%	0.89%

Since a temperature of 90° F. will seldom or never prevail at an altitude where the pressure is as low as 20 inches, we may regard 1 per cent. as about the maximum possible error, and in the vast majority of cases the actual error will be less than 0.5 per cent. Now with the anemometers we need to consider, a given percentage error in the density causes only about half as much error in the speed S ; and furthermore, an accuracy of 1 per cent. in measuring the speed of an aeroplane may be regarded as satisfactory. Hence the assumption of a constant relative humidity of 50 per cent. ($H=0.5$) is quite approximate enough for our purpose, and we adopt this assumption and thereby simplify equation (6) to the form

$$\rho = 1.327 \frac{B - 0.19 P}{460 + t} \text{ pounds per cubic foot.} \quad (8)$$

From equation (8) we may now compute a table of approximate values of the air density at various values of the barometric pressure B and the temperature t . It will be convenient to have the values expressed, not in pounds per cubic foot, but in terms of a standard air density, and for this the value $=0.07455$ has been chosen. This is the density at $B=29.92$ inches, $t=70^\circ$ F., and $H=0.5$, conditions which are a fair average representation of those which are likely to prevail during anemometer tests. The values are shown in Table 7.

TABLE 7.—Relative density D of air at B inches pressure, t° F., and 50 per. cent relative humidity, referred to air at 29.92 inches pressure, 70° F., and 50 per. cent. relative humidity.

$B=$	20"	22"	24"	26"	28"	30"
$t=0^\circ$ F.	0.773	0.851	0.928	1.006	1.083	1.160
10°757	.833	.908	.984	1.060	1.135
20°741	.815	.889	.963	1.037	1.112
30°725	.798	.871	.943	1.016	1.088
40°710	.781	.853	.924	.995	1.066
50°696	.766	.835	.905	.975	1.045
60°681	.750	.818	.887	.955	1.023
70°667	.734	.801	.868	.935	1.003
80°653	.719	.785	.850	.916	.982
90°639	.703	.768	.833	.897	.962

We have next to consider how these variations of density may affect the readings of an anemometer which has been tested under standard conditions.

A. *The Pitot tube.*—The Pitot tube formula may be written

$$S = \text{const} \times \sqrt{\frac{p_1 - p_2}{\rho}}$$

or for a standard density ρ_0

$$S_0 = A_0 \sqrt{p_1 - p_2}$$

At any other density, $\rho = D\rho_0$, we have

$$S = \frac{A_0}{\sqrt{D}} \sqrt{p_1 - p_2} = \frac{S_0}{\sqrt{D}} \quad (9)$$

If the tube has been standardized at the density ρ_0 and the constant A_0 determined, or if the gage has been provided with a speed scale or a table for converting its readings at the standard density ρ_0 into speeds, the true speed at any other density ρ is found by multiplying the indicated speed by $\frac{1}{\sqrt{D}}$. Values of $\frac{1}{\sqrt{D}}$ computed from Table 7 are given in Table 8.

TABLE 8.—Values of $\frac{1}{\sqrt{D}}$ for use in equation (9).

t° F.	Barometric height <i>B</i> in inches of mercury.					
	20"	22"	24"	26"	28"	30"
0.....	1.137	1.084	1.038	0.979	0.961	0.928
10.....	1.149	1.096	1.049	1.008	.971	.938
20.....	1.162	1.108	1.061	1.019	.982	.948
30.....	1.174	1.119	1.072	1.030	.992	.958
40.....	1.187	1.131	1.083	1.040	1.003	.968
50.....	1.199	1.143	1.094	1.051	1.013	.978
60.....	1.212	1.155	1.106	1.062	1.023	.989
70.....	1.225	1.167	1.117	1.073	1.034	.999
80.....	1.238	1.180	1.129	1.084	1.045	1.009
90.....	1.251	1.193	1.141	1.096	1.056	1.020

If the purpose of reading the anemometer is not, primarily, to ascertain the speed, but to judge of the wind pressures on the machine which determine the lift and the stresses, then the density correction should *not* be applied. For at any given angle of attack, the wind forces are very nearly proportional to the Pitot pressure; when the gauge shows a given reading, the wind forces are always the same; and from the standpoint of sustaining power and strength it is immaterial how the forces arise. Hence from the point of view of the aviator who is concerned with the safety of his machine, the

speed readings of the Pitot-tube anemometer correct themselves automatically—if the machine flies safely at a given speed and in air of a given density, it will be equally safe in air of any other density, regardless of pressure, temperature, and humidity if the Pitot-tube gauge gives the same reading.

B. *Pressure-plate anemometers*.—It would naturally be supposed that the readings of pressure plate anemometers would be affected by variations of air density in the same way as those of Pitot tubes. The theory of the subject, however, is not entirely clear, and it is difficult to interpret some of the experimental results which have been obtained.¹ In the absence of further investigation it would seem safest to make the density correction, when necessary, exactly as is done for the Pitot tube. If the readings are taken only for the sake of estimating the wind forces on the machine, the density correction is to be omitted, just as with the Pitot tube.

C. *The Bourdon-Venturi anemometer*.—If the results of Bourdon's experiments agreed closely with computations from the theoretical equation of the Venturi meter, we should feel justified in using that equation to compute density corrections to be applied to the readings of an instrument which had been tested at a standard air density. But the discrepancies shown by curves GH and EF of figure 1 are so large that we can not trust the theoretical equation at all for a Venturi tube used as an anemometer. It appears that further experimental investigations of this instrument are needed.

D. *Rotary anemometers*.—Regarding rotary anemometers, Jones and Booth² say:

The principal advantage possessed by instruments of this type is that they read the actual travel through the air independently of variations in density.

It seems likely, however, that this independence is only approximate and not complete. The ratio of cup or vane speed to wind speed depends on the value of the least wind speed which will just keep the anemometer turning against friction. And since each vane or cup when moving very slowly acts as a pressure plate, it seems that the wind speed required in order to furnish the torque for very low speeds of rotation must depend on the air density. Hence it seems probable that at higher speeds the action of instruments of the Robinson or of the screw type is somewhat influenced by air density. Exact information on this is lacking.

14. COMPARISON OF TYPES OF ANEMOMETER.

Anemometers in general might be compared from various points of view; but since our purpose is strictly practical, we shall at once exclude from the discussion any instrument which can not be made satisfactory on the score of (a) robustness combined with lightness, (b) independence of gravity, and (c) flexibility of transmission, permitting the head to be placed at a distance from the indicator in front of the pilot's seat. There seem then to remain for discussion the Pitot tube, the pressure plate, the Venturi tube, and the Robinson anemometer.

A. *The Pitot tube*.—This has been the most studied, and we can speak of it with more certainty than of the others. The head is

¹ See Rayleigh, Rep. Brit. Adv. Com. for Aeronautics, 1910-11, p. 26.

² Aeronautical Journal, July, 1913, p. 192.

simple and may be placed in any position; and the transmission of the pressure through tubes presents no obvious difficulties. The prime defect of the instrument is the smallness of the pressure available for actuating the indicator. While sensitive liquid gauges may be used under some circumstances, anything but a spring gauge seems out of the question for all-round use. The problem with the Pitot tube is to make a satisfactory spring gauge which shall at the same time be sufficiently sensitive and so robust as to be reliable. The problem looks difficult, but may not be insoluble.

B. *The pressure plate.*—By an increase of size, the pressure plate may be made to give as large a force as is desired, the limit being set by the amount of head resistance which it is considered permissible to devote to an anemometer. Transmission by wires under tension might be practicable but would be liable to get out of order and to be seriously disturbed by vibration. Transmission by means of liquid pressure might be managed but would introduce complications, and the development of the instrument in this form would demand a great deal of experimentation. In spite of its attractiveness and apparent simplicity at first sight, the pressure plate does not, on the whole, seem very promising as a practical aeroplane instrument.

C. *The Bourdon-Venturi anemometer.*—The Venturi tube furnishes a pressure difference and the transmission problem is simple, as it is with the Pitot tube. But the pressure difference may be made so large that the problem of making a satisfactory spring gauge is vastly simpler than with the Pitot tube, and should not present any insuperable difficulties. A more important doubt arises in connection with the density correction. Since it is impracticable to test an anemometer at low-air densities by the ordinary methods, and since Bourdon's results differed greatly from what might have been expected on theoretical grounds, the instrument should be used with caution, if high altitude flights are in question, until we know more about its practical behavior. On the other hand, it appears to be satisfactory at ordinary air densities,¹ and it seems to be an instrument of great promise and one of which the practical development should be pushed along.

D. *The Robinson anemometer.*—The weak point of the Robinson anemometer is lack of flexibility in the transmission. In the form of Morell's anemo-tachometer it indicates speed through the air nearly independently of the air density. But since the main purpose of knowing this speed is for finding the total distance traveled, it would seem as if the ordinary method of registering the total number of turns would, in practice, be more useful than the attachment of a tachometer to give instantaneous speeds.

Having now discussed some of the mechanical characteristics of the four types of instrument we may take another standpoint and, assuming that a mechanically satisfactory instrument of each type can be constructed, ask whether one presents any advantages over another. The answer to this question depends on why we want to know the speed.

If what is wanted is to estimate the distance traveled through the air, some form of Robinson anemometer seems to be the thing to use, because it is independent of air density, to a first approximation, at

¹ See Eiffel, *The Resistance of the Air*, p. 234.

all events. The other three types of instrument will all require to have a density correction applied to their readings, if the air density is far different from that during standardization, and they are thus at a disadvantage.

But it appears that the speed through the air is, in general, not itself the important quantity sought; for at best it does not tell us the speed over the ground until it is compounded with the speed of the wind which may happen to be blowing. A more important use of the anemometer is not properly as a speedometer but as a dynamometer, i. e., as an instrument for indicating the air forces on the machine. For this purpose, any instrument such as the anemotachometer which gives the speed without reference to the density will require a density correction to its readings, whereas the Pitot tube gives just what is wanted, the allowance for density being already present in its uncorrected readings, so that equal readings mean equal pressures, whatever the density may be. The pressure plate falls in the same class as the Pitot tube. Of the Bourdon-Venturi anemometer we can say very little until the instrument has been further studied, but it seems likely that it also will act rather as a dynamometer than as a speedometer, if its readings are not corrected for variations of air density.

Still another question which may be asked is, What sort of mean speed does a given anemometer indicate when exposed to a gusty wind? In regard to this question, the four types under consideration fall into the same grouping as before. With the Pitot tube, the pressure plate, or the Venturi tube, the pressure difference or the force depends on the square of the wind speed, and the mean reading of any of these instruments in a wind of varying speed will therefore give not the arithmetical mean speed but the root-mean-square speed, which is what determines the mean wind forces on the aeroplane. The anemotachometer, on the other hand, will probably indicate something between the arithmetical mean and the root-mean-square speed. If it had no inertia it might be made to indicate the arithmetical mean, but the effects of inertia in causing lag or lead will probably make the mean reading of the instrument in a wind of variable strength somewhat higher than it would be in the absence of inertia. The fact that this might result in a slight overestimate of the total travel will hardly be of any moment, in view of the impossibility, for the aviator, of measuring and allowing for the true velocity of the wind with respect to the earth's surface.

REPORT No. 2.

PART 2.

THE THEORY OF THE PITOT AND VENTURI TUBES.

By E. BUCKINGHAM.

1. THE ENERGY EQUATION FOR STEADY ADIABATIC FLOW OF A FLUID.

Let a fluid be flowing steadily along a channel with impervious and nonconducting walls, from a section A to a section A_1 , the areas of the sections perpendicular to the direction of flow being also denoted by A and A_1 . By saying that the flow is "steady" we do not mean that it occurs in stream lines and without turbulence. We mean merely that it is "sensibly" steady; i. e., that such variations of speed, direction of motion, pressure, etc., as may occur at any point in the stream as a result of turbulence are so rapid that our measuring instruments do not respond to them, but indicate only time averages; and that these time averages are constant at any fixed point within the channel. Values of a property of the fluid, or of any other quantity such as speed, "at a point," are therefore to be understood as time averages over a time which is long compared with the speed of variation of the quantity to be measured, though it may appear short in the ordinary sense.

Let θ , p , v , ϵ , T , respectively, be the absolute temperature, static pressure, specific volume, internal energy per unit mass, and kinetic energy per unit mass, at the entrance section A . By the "static pressure" is meant the pressure which would be indicated by a gauge moving with the current. Let θ_1 , p_1 , v_1 , ϵ_1 , T_1 be the corresponding quantities at the exit section A_1 . Both sets of values are to be understood as averages over the whole section, as well as time averages in the sense explained above. The two sections shall be at the same level, so that the passage of fluid from A to A_1 does not involve any gravitational work.

As a unit mass of fluid crosses A , the work pv is done on it by the fluid following; and as it crosses A_1 it does the work p_1v_1 on the fluid ahead. Since the walls of the channel are nonconducting, no heat enters or leaves the fluid between A and A_1 ; hence the total energy, internal plus kinetic, increases (or decreases) by an amount equal to the work done on (or by) the fluid, and we have

$$\begin{aligned} pv - p_1v_1 &= (\epsilon_1 + T_1) - (\epsilon + T) \\ T - T_1 &= (\epsilon_1 + p_1v_1) - (\epsilon + pv) \end{aligned} \tag{1}$$

So far no assumptions have been made and equation (1) is rigorously correct for adiabatic flow between two sections at the same level. Internal heating by skin friction or the dissipation of eddies is merely a conversion of energy from one form into another and not an addition of energy; hence it does not affect the validity of equation (1) and need not appear in it.

2. INTRODUCTION OF THE MEAN SPEED INTO THE ENERGY EQUATION.

Let Q be the volume of fluid which crosses the section A per unit time, and let $S = Q \div A$; then S is the arithmetical mean, over the section, of the component velocity normal to A and along the channel. Let Q_1 and S_1 be the corresponding values at A_1 . Measuring kinetic energy, as well as work and internal energy, in normal mass-length-time units, we then set

$$T - T_1 = \frac{1}{2} (S^2 - S_1^2) \quad (2)$$

and proceed to substitute this expression for $(T - T_1)$ in equation (1).

This substitution is indispensable to further progress, but it involves an assumption which destroys the rigor of all further deductions. The deductions are, nevertheless, very approximately confirmed by experiment, and it is therefore worth while to examine the assumption.

If there were no turbulence and if the speed were uniform over each section, we should have the two separate equations

$$T = \frac{1}{2} S^2 \quad (3)$$

$$T_1 = \frac{1}{2} S_1^2$$

and equation (2) would be exact. If there is no turbulence but the speed of flow is nonuniform, approaching zero at the walls, as it must where the channel has material walls, equations (3) will not be satisfied, but we shall have $T > \frac{1}{2} S^2$ and $T_1 > \frac{1}{2} S_1^2$, because the mean square speed, which determines the kinetic energy, is always greater than the arithmetical mean speed S when the distribution over the section is not uniform. With a round pipe and nonturbulent flow $T = \frac{3}{8} S^2$ instead of $\frac{1}{2} S^2$.

In nearly all practical cases the flow of fluids is turbulent and the relation of the whole kinetic energy, including that of the turbulence, to the arithmetical mean normal component of the speed at the given section will depend on the amount of turbulence. It is impossible to say what the relation will be further than that the kinetic energy of eddies and cross currents tends to increase the error which would be involved in assuming equations (3), while, on the other hand, the fact that with increasing turbulence the speed becomes more nearly uniform over a cross section tends to decrease the difference between the mean square and the arithmetical mean of the component normal to any section.

The assumption involved in using equation (2) is not, however, so violent as that which would be involved in using equations (3) separately. For equations (3) are equivalent to

$$T - \frac{1}{2}S^2 = T_1 - \frac{1}{2}S_1^2 = 0$$

whereas equation (2) is satisfied if

$$T - \frac{1}{2}S^2 = T_1 - \frac{1}{2}S_1^2 \quad (4)$$

no matter what the value is. Equation (4) and its equivalent (2) are satisfied if the error in assuming equations (3) to hold is the *same* at both sections without vanishing or even being small. This will occur if the kinetic energy of turbulence is the same at both sections and if also the speed distributions over the two sections are such that the arithmetical mean normal speed is the same fraction of the mean-square normal speed at both. While therefore it is evident that the use of equations (3) separately might lead to conclusions at variance with facts, equation (2) may nevertheless be nearly fulfilled in practice. The agreement with observation of deductions from equations (2) and (1) shows that in many ordinary cases the error committed by treating equation (2) as exact is in reality quite insignificant.

For geometrically similar channels, the percentage error of equation (2) depends only on $\frac{DS}{\nu}$, in which ν is the kinematic viscosity of the fluid and D a linear dimension of the channel. With a given fluid in a given channel increasing S increases the turbulence, but it is not evident how this will affect the percentage error, $\frac{2T - S^2}{S^2}$, if at all. Hence, it seems possible that although turbulence increases with $\frac{DS}{\nu}$, the *percentage* error in assuming equation (2) may not increase but remain constant or even decrease. On the other hand, at a given speed S , if $\frac{DS}{\nu}$ is increased by increasing D or diminishing ν , the turbulence and the value of $\frac{2T - S^2}{S^2}$ will be increased

and there will be a greater chance that equation (2) may be sensibly in error. At a given mean axial speed S we must therefore be prepared to find greater discrepancies between experiment and results deduced from equation (2) for large channels and fluids of low kinematic viscosity than for the opposite conditions.

We shall now proceed as if equation (2) were rigorously exact, and by combining it with equation (1) we obtain

$$\frac{1}{2}(S^2 - S_1^2) = (\epsilon_1 + p_1 v_1) - (\epsilon + p v) \quad (5)$$

an equation which serves as the point of departure for the theory of the Pitot tube, the Venturi meter, the steam-turbine nozzle, and various other devices in which a stream of fluid is retarded or accelerated adiabatically.

3. ISENTROPIC FLOW OF AN IDEAL GAS.

If the physical properties of the fluid have been sufficiently investigated and if a sufficient number of quantities are measured at each of the two sections, the value of $(\epsilon + pv)$ may be computed for each section and the value of $(S^2 - S_1^2)$ found from equation (5), to the degree of approximation permitted by the assumptions which have been discussed above. A process somewhat of this nature is pursued in the design of steam-turbine nozzles, $(\epsilon + pv)$ being then the quantity known as the total heat of steam.

But when the fluid is a gas, it is usual to proceed with deductions from equation (5) by the aid of two further assumptions which enable us to compute variations of ϵ and v from observations of p alone. The first of these assumptions is that the fluid behaves sensibly as an ideal gas defined by the equations

$$pv = R\theta \quad (6)$$

$$\epsilon = \epsilon_0 + C_v (\theta - \theta_0) \quad (7)$$

in which C_v is the specific heat at constant volume, and ϵ_0 is the internal energy at the standard temperature θ_0 . The properties of ordinary gases, such as air, carbon dioxide, or coal gas, when far from condensation, are nearly in conformity with equations (6), and (7), and for such fluids no serious error is involved in making the assumption mentioned, unless very great variations of pressure and temperature are under consideration. Equations (6) and (7) imply also the relation

$$C_p = C_v + R \quad (8)$$

in which C_p is the specific heat at constant pressure.

The second assumption is that during the simultaneous changes of pressure and temperature in passing from A to A_1 the familiar isentropic relation for an ideal gas, viz,

$$\frac{\theta_1}{\theta} = \left(\frac{p_1}{p} \right)^{\frac{k-1}{k}} \quad (9)$$

remains satisfied, k representing C_p/C_v . This assumption is, of course, not exact, for while we have stipulated that the flow shall be adiabatic, the internal heating, due to viscosity causes an increase of entropy. The assumption amounts, therefore, to assuming that this irreversible internal heating is not enough to cause any sensible increase of the temperature at A_1 over what it would be if there were no internal heating at all.

The foregoing assumptions enable us to put equation (5) into a more available form. By substituting from (6) and (7) into (5), and using (8), we have

$$\frac{1}{2} (S^2 - S_1^2) = C_p (\theta_1 - \theta) \quad (10)$$

By means of (9) and (6), this may be written

$$\frac{1}{2} (S^2 - S_1^2) = \frac{C_p}{R} pv \left[\left(\frac{p_1}{p} \right)^{\frac{k-1}{k}} - 1 \right]$$

and by (8) we get $C_p/R = \frac{k}{k-1}$ so that we have

$$\frac{1}{2} (S^2 - S_1^2) = \frac{k}{k-1} p v \left[\left(\frac{p_1}{p} \right)^{\frac{k-1}{k}} - 1 \right] \quad (11)$$

which is the usual form of equation (5) for isentropic flow of an ideal gas. If the speed is known at either section, equation (10) enables us to find the speed at the other from a knowledge of C_p and an observation of the difference of temperature; while equation (11) gives us similar information in terms of the pressures at A and A_1 if the density and the ratio k are known. We shall apply this equation to both the Pitot tube and the Venturi meter.

4. THE THEORY OF THE PITOT TUBE.

To treat the Pitot tube, we consider the fluid which is approaching the dynamic opening. Starting at a point so far upstream that the presence of the Pitot tube produces no sensible disturbance there, a particle of fluid approaches the dynamic opening, slows down, and mixes with the permanent high-pressure cap of nearly stationary fluid, which covers the dynamic opening and communicates with the differential gauge through the impact tube. The same particle, or another indistinguishable from it, emerges from the cap and, being accelerated by the now positive pressure gradient, flows on along the impact tube, finally acquiring a sensibly constant speed when it has reached a region of sensibly constant pressure. We wish to apply equation (5) to this motion if we can find a plausible way of doing so.

Starting with the contour of a small plane area, in the undisturbed current and perpendicular to its general direction, we construct, in imagination, a tubular surface of which the sides are at every point parallel to the mean direction of motion of the fluid past that point, as found by averaging with regard to time. If the motion is not turbulent, this tube is a tube of flow and no fluid passes in or out through its sides. If the motion is turbulent, as it nearly always is in practice, the *same* fluid does not flow continuously along the tube as it would if the walls were impervious. On the contrary, particles of fluid are continually leaving the tube in consequence of the turbulent time-changes of the direction of motion at any fixed point; and these particles are continually replaced by others, of the same total mass, which enter from without the tube. But on the whole, the particles which enter have the same average component velocity along the tube as those which leave; for unless this were true we could, merely by *imagining* the tubular surface, generate within the fluid a particular filament which was moving, on the whole, faster or slower than the surrounding fluid. We conclude that the net effect of turbulence is the same as if the imaginary tube walls were made rigid and perfectly reflecting for mechanical impact without exerting any skin friction on the fluid flowing along them.

If the whole current of fluid is at a sensibly uniform temperature across its general direction, no heat passes in or out through the tubular surface, and equation (5) may be applied as though we had an impervious nonconducting channel to deal with. Furthermore, if the tube is of small section, the axial speed, averaged with regard

to time, will be the same at all points of any one cross section. Hence the application of equation (5), involving the assumption of equation (2) or (4), is better justified than for a material tube in which skin friction would cause the axial speed to be nonuniform over any section.

We now consider such an imaginary tube, starting in the undisturbed fluid some distance upstream from the dynamic opening of the Pitot tube, passing into the high-pressure cap over the opening and emerging again at the edge of the opening, to continue its course along the side of the impact tube. The portion of the imaginary tube which passes through the high-pressure cap may be regarded as an enlargement of cross section at which the mean axial speed is so reduced that its square is negligible in comparison with the square of the speed at distant points. If we let A be a section at some distance upstream and A_1 be the section of the tube where it passes through the high-pressure cap, S_1^2 is negligible in comparison with S^2 and equation (5) gives us

$$S = \sqrt{2[(\epsilon_1 + p_1 v_1) - (\epsilon + p v)]} \quad (12)$$

in which S is the speed of the undisturbed current; ϵ , p , and v refer to conditions in the undisturbed current; and ϵ_1 , p_1 , v_1 refer to conditions in the dynamic opening. The static pressure, which the static opening is designed to receive and transmit to the gauge, is p ; while the pressure received by the dynamic opening is that in the permanent high-pressure cap, or p_1 .

Equation (12) is the general form of the Pitot tube equation for any fluid, whether compressible or not. In the case of a liquid, the internal energy and specific volume are not appreciably affected by the very small pressure variations involved, so that we have $\epsilon_1 = \epsilon$ and $v_1 = v$ and equation (12) reduces to

$$S = \sqrt{2v(p_1 - p)} = \sqrt{2 \frac{p_1 - p}{\rho}} \quad (13)$$

ρ being the density of the liquid. If the pressure difference is expressed as a head h of liquid of density d , we have $p_1 - p = gh d$ and equation (13) takes the form

$$S = \sqrt{2g \frac{d}{\rho} h} \quad (14)$$

the usual form of the Pitot tube equation for a perfect or ideal tube.

Even when the fluid is a gas, if S is small and $(p_1 - p)$ therefore also small, ϵ_1 and v_1 are nearly the same as ϵ and v so that equations (13) and (14) remain approximately correct—admitting all the assumptions made—though it is not evident how close the approximation will be. But if the speed and the pressure difference are great enough to cause sensible compression, we must return to equation (5) and introduce the conditions for adiabatic flow of a gas, as was done in section 3 in arriving at equation (11). The fact that equation (14) does agree well with observations on gas currents at moderate speeds, shows that no great error is involved in neglecting compressibility

and justifies us in going on to find a closer approximation by treating the gas as ideal and thereby using an approximation to the compressibility.

Assuming, then, that equation (11) is applicable to the imaginary current tube now under discussion, we have, by setting $S_1^2 = 0$, the equation

$$S = \sqrt{\frac{2k}{k-1} \frac{p}{\rho} \left[\left(\frac{p_1}{p} \right)^{\frac{k-1}{k}} - 1 \right]} \quad (15)$$

If we now set $\frac{p_1}{p} = 1 + \Delta$ and $\frac{k-1}{k} = n$ we have

$$\left(\frac{p_1}{p} \right)^{\frac{k-1}{k}} - 1 = n\Delta \left\{ 1 + \frac{n-1}{2} \Delta + \frac{(n-1)(n-2)}{1 \cdot 2 \cdot 3} \Delta^2 + \text{etc.} \right\}$$

Setting the $\{ \dots \} = X^2$, substituting in equation (15), and noticing

that $n\Delta = \frac{k-1}{k} \frac{p_1 - p}{p}$ we have

$$S = X \sqrt{2 \frac{p_1 - p}{\rho}} \quad (16)$$

which differs from equation (13), obtained by disregarding compressibility, only in the correction factor

$$X = \left\{ 1 + \frac{n-1}{2} \Delta + \frac{(n-1)(n-2)}{1 \cdot 2 \cdot 3} \Delta^2 + \frac{(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3 \cdot 4} \Delta^3 + \dots \right\}^{\frac{1}{2}} \quad (17)$$

The quantity $\Delta = \frac{p_1 - p}{p}$ is the fractional rise of pressure at the mouth of the impact tube: hence it is, in practice, always a small quantity. The value of k for gases is always between $\frac{5}{3}$ and 1, so that $n = \frac{k-1}{k}$ is always between $\frac{2}{3}$ and 0. Accordingly the terms of X containing Δ are alternately negative and positive and when Δ is small the series converges rapidly, the sum of all the terms in Δ being nearly equal to the first term alone, so that if the first is negligible the sum is negligible and X may be set equal to unity.

The ratio of the specific heats of air is 1.40. Hence $n = \frac{2}{7}$ and we have

$$X = \left\{ 1 - \frac{5}{14} \Delta + \frac{10}{49} \Delta^2 - \frac{95}{686} \Delta^3 + \text{etc.} \right\}^{\frac{1}{2}} \quad (18)$$

If an error of y per cent. in S is permissible, an error of y per cent. may also be allowed in the correction factor X and the value of Δ may be, at most, such as to make $\frac{5}{28} \Delta = \frac{y}{100}$ or $\Delta = 0.056y$. For any assigned values of the error y per cent. in the speed, the value of S can be found from equation (13).

Let us suppose, for example, that the Pitot tube is to be used for measuring the speed of an aeroplane and that an accuracy of 0.5 per cent. is sufficient. Then we have $\Delta = 0.028$ and $p_1 - p = 0.028 p$. To find what speed would give this head on the differential gauge, we set $p = 1$ atmosphere $= 1.013 \times 10^6$ dynes/cm.² and $\rho = 0.0013$ gram/cm.,³ and substitute in (13), the result being $S = 66.1$ m./sec. $= 212$ ft./sec. $= 148$ miles/hour. Since an accuracy of better than 1.0 per cent. can hardly be demanded of an aeroplane speedometer, it is evident that for all ordinary speeds of flight, no correction for compressibility is needed and equations (13) and (14) may be used.

It is of course a simple matter to compute values of the correction factor X for various speeds; but in view of the uncertainties and assumptions involved in the theory, the results would have a misleading appearance of accuracy and would not in fact be worth the labor of computation. What has been shown is sufficient, namely, that if a Pitot tube does not measure the speed of an aeroplane correctly the error is not due to neglecting the compressibility of the air.

5. THE THEORY OF THE VENTURI METER.

The Venturi meter is a channel of varying cross section, and we may apply to it the general equations of flow which have already been developed. In doing so, we shall let A be the entrance section of the meter where p is measured, and A_1 be the throat section at which the diminished pressure p_1 is observed. We have to use equation (5).

If the meter is used for measuring the flow of a liquid of density ρ we may set $\epsilon_1 = \epsilon$ and $v_1 = v$ as we did in treating the Pitot tube, and equation (5) then gives us

$$S_1^2 - S^2 = 2 \frac{p - p_1}{\rho} \quad (19)$$

Neither S nor S_1 vanishes; but in addition to (19) we have the equation of continuity which for a fluid of constant density may be written

$$S_1 A_1 = S A \quad (20)$$

and (19) and (20) together enable us to find either S or S_1 . If we represent the area ratio by a single symbol

$$\frac{A}{A_1} = \alpha > 1 \quad (21)$$

we have

$$S = B \sqrt{2 \frac{p - p_1}{\rho}} \quad (22)$$

where

$$B = \sqrt{\frac{1}{\alpha^2 - 1}} \quad (23)$$

and B is a constant characteristic of the given meter.

Comparing (22) with (13), the equation for the Pitot tube in a liquid, we see that they differ only by the factor B which depends on

the area ratio α . If $\alpha = \sqrt{2}$, $B = 1$ and the observed Venturi pressure difference $(p - p_1)$ will be the same as would be shown by a Pitot tube with its dynamic opening in the entrance of the meter. For various values of the ratio $\frac{D}{D_1}$ of entrance diameter to throat diameter we have the following values of B :

$\frac{D}{D_1} =$	1.5	2.0	2.5	3.0	4.0
$\alpha =$	2.25	4.00	6.25	9.00	16.00
$B =$	1.569	3.874	6.170	8.944	15.77

Evidently, the Venturi pressure difference may easily be made much larger than the Pitot pressure difference at the entrance speed and the gauge reading be made much more sensitive.

If the fluid is a gas instead of a liquid, compressibility will still be negligible at sufficiently low speeds, as for the Pitot tube, and equation (22) may be used; but in general the compressibility must be allowed for. To treat the flow of a gas, we have to make the same assumptions as in section 3, namely, that the gas is sensibly ideal and that the flow from the entrance section A to the throat A_1 is sensibly isentropic, the combined effect of heat conduction to or from the walls of the meter, and of internal heating in the gas itself, being insignificant. We then have to apply equation (11) to the case in hand, and if for simplicity we represent the pressure ratio by a single symbol and write

$$\frac{p_1}{p} = r < 1 \quad (24)$$

we have by equation (11)

$$S_1^2 - S^2 = \frac{2k}{k-1} \frac{p}{\rho} \left[1 - r^{\frac{k-1}{k}} \right] \quad (25)$$

ρ being the density of the gas at the pressure p as it crosses the entrance section.

To combine with (25) we have the equation of continuity

$$S_1 A_1 \rho_1 = S A \rho$$

and if we remember that during isentropic compression or expansion of an ideal gas pv^k remains constant, the equation of continuity may be written

$$S_1 = \frac{\alpha}{r^{1/k}} S \quad (26)$$

By using (26) to eliminate S_1 from (25) we now obtain the equation

$$S = \left\{ \frac{2k}{k-1} \frac{r^{2/k}}{\alpha^2 - r^{2/k}} \frac{p}{\rho} \left(1 - r^{\frac{k-1}{k}} \right) \right\}^{1/2} \quad (27)$$

by means of which the entrance speed S may be computed from the observed pressure ratio $r = p_1/p$ when the area ratio α and the properties of the gas are known. Since we are treating the gas as

ideal, p/ρ is, for any given gas, proportional to the absolute temperature θ at the entrance section, and we may write $\frac{p}{\rho} = \frac{p_o}{\rho_o} \frac{\theta}{\theta_o}$, ρ_o being the density of the gas at the standard pressure p_o and temperature θ_o .

For air, $\frac{C_p}{C_v} = k = 1.40$ and if we insert the known value of ρ_o at 1 atmosphere and 0° C. and set

$$S = Y \sqrt{\frac{\theta}{\theta_o}} \quad (28)$$

where

$$Y = \left\{ \frac{2k}{k-1} \cdot \frac{r^{2/k}}{\alpha^2 - r^{2/k}} \left(1 - r^{\frac{k-1}{k}} \right) \frac{p_o}{\rho_o} \right\}^{1/2}$$

we have the values of Y shown in the following table for various pressure ratios r and for meters in which the throat diameter is $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ of the entrance diameter, i. e., $\alpha = 4, 9$, or 16 . If t is the temperature at entrance, on the centigrade scale $\frac{\theta}{\theta_o} = \frac{273+t}{273}$ while if t is measured on the Fahrenheit scale,

$$\frac{\theta}{\theta_o} = \frac{460+t}{492}$$

THE VENTURI METER FOR AIR.

Values of Y in $S = Y \sqrt{\frac{\theta}{\theta_o}}$

S = Speed at entrance to meter $\alpha = \frac{A}{A_1} = \frac{\text{entrance area}}{\text{throat area}}$

r = throat pressure \div entrance pressure = p_1/p θ = absolute temperature of air at entrance.

θ_o = absolute temperature of ice point.

Values of Y .

r	$\alpha=4$			$\alpha=9$			$\alpha=16$		
	M./sec.	Ft./sec.	Mile/hour.	M./sec.	Ft./sec.	Mile/hour.	M./sec.	Ft./sec.	Mile/hr.
0.9998	1.44	4.74	3.23	0.626	2.05	1.400	0.350	1.150	0.784
.999	3.23	10.60	7.23	1.40	4.59	3.13	0.784	2.57	1.753
.995	7.21	23.65	16.13	3.12	10.24	6.98	1.75	5.74	3.91
.99	10.16	33.34	22.7	4.40	14.11	9.85	2.47	8.09	5.52
.98	14.3	46.48	32.0	6.19	20.3	13.85	3.47	11.38	7.76
.95	22.2	72.8	49.6	9.62	31.6	21.5	5.39	17.7	12.06
.90	30.4	99.8	68.0	13.2	43.4	29.6	7.41	24.3	16.57
.80	40.2	131.7	89.8	17.5	57.5	39.2	9.82	32.2	22.0
.60	48.1	157.9	107.6	21.1	69.3	47.2	11.86	38.9	26.5

Computed on the assumptions $p_v = R\theta$, $C_v = \text{constant}$, $\frac{C_p}{C_v} = 1.400$.

$p_o = 1.01323 \times 10^6$ dyne/cm².

$\rho_o = 0.0012928$ gm cm³ at 760 mm. and 0° C